



Sequence & Series Assignment -1

- 1.) Show that $(x^2 + xy + y^2)$, $(z^2 + xz + x^2)$ and $(y^2 + yz + z^2)$ are consecutive terms of an A.P., if x, y and z are in A.P.
- 2.) a) The third term of G.P. is 4. Find the product of its first 5 terms.
b) In a G.P, the first term is 7, last term is 448 & the sum is 889, Find the common ratio & number of terms.
- 3.) Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7^{th} and $(m - 1)^{\text{th}}$ numbers is 5 : 9. Find the value of m.
- 4.) If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.
- 5.) If p^{th} , q^{th} , r^{th} and s^{th} terms of an A.P. are in G.P, then show that $(p - q)$, $(q - r)$, $(r - s)$ are also in G.P.
- 6.) Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.
- 7.) The A.M b/w two numbers a & b is twice the G.M, Prove that ; $a : b = (2 + \sqrt{3}) : (2 - \sqrt{3})$.
- 8.) Find the sum of n terms of sequence : $0.6 + .06 + .006 + \dots \dots \dots n$ terms.
- 9.) If p, q, r are in G.P & the equations $px^2 + 2qx + r = 0$ & $dx^2 + 2ex + f = 0$ have a common root, then show that, d/p , e/q , f/r are in A.P.
- 10.) If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that $\frac{(q + p)}{(q - p)} = \frac{17}{15}$.
- 11.) If f is a function satisfying $f(x + y) = f(x) f(y)$ for all $x, y \in \mathbb{N}$ such that $f(1) = 3$ and $\sum_{x=1}^n f(x) = 120$, Find the value of n.
- 12.) Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{\text{th}}$ to $(2n)^{\text{th}}$ term is $\frac{1}{r^n}$.
- 13.) If $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P, Show that either a, b, c are in A.P or $ab + bc + ca = 0$.
- 14.) Find the value of n so that $[a^{(n+1)} + b^{(n+1)}] / [a^n + b^n]$ may be the geometric mean between a and b.
- 15.) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.
- 16.) Find the value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/6} \dots \dots \dots \infty$
- 17.) There are n arithmetic means between 3 & 17. The ratio of the last mean to the first mean is 3 : 1, Find 'n'.
- 18.) If $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P, Show that either a, b, c are in A.P or $ab + bc + ca = 0$.
- 19.) Find the sum of 50 terms of sequence : $7 + 7.7 + 7.77 + 7.777 + \dots \dots \dots 50$ terms
- 20.) The sum of first two terms of an infinite geometric series is 15 & each term is equal to the sum of all the terms following it. Find the series.

- 21.) If p, q, r are in A.P while x, y, z are in G.P, prove that : $x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = 1$.
- 22.) The sum of two numbers is $13/6$. An even number of A.Ms. are being inserted between them & their sum exceeds their number by 1, Find the number of means inserted.
- 23.) Let ratio of the A.M & the G.M of two +ve numbers a & b be $m : n$, show that : $a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2})$
- 24.) The A.M b/w two numbers a & b is twice the G.M, Prove that ; $a : b = (2 + \sqrt{3}) : (2 - \sqrt{3})$.
- 25.) The sum of three numbers which are in G.P is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an Arithmetic Progression. Find these numbers.
- 26.) The sum upto n terms of the series : $1 + (1+x)y + (1+x+x^2)y^2 + (1+x+x^2+x^3)y^3 + \dots$
- 27.) Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2 R^n = S^n$.
- 28.) Find the value of n so that $[a^{(n+1)} + b^{(n+1)}] / [a^n + b^n]$ may be the geometric mean between a and b .
- 29.) Prove that for two positive no. $A.M > G.M$. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.
- 30.) If the first and the n th term of a G.P. are a and b , respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$
- 31.) Find the sum to n terms of the sequence, 8, 88, 888, 8888,
- 32.) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.
- 33.) If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.
- 34.) Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.
- 35.) If p, q, r are in A.P while x, y, z are in G.P, prove that : $x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = 1$.
- 36.) The A.M b/w two numbers a & b is twice the G.M, Prove that ; $a : b = (2 + \sqrt{3}) : (2 - \sqrt{3})$.
- 37.) There are n arithmetic means between 3 & 17. The ratio of the last mean to the first mean is 3 : 1, Find 'n'.
- 38.) The ratio of the sum of m & n terms of an A.P is $m^2 : n^2$. Show that the ratio of m^{th} & n^{th} term is $2m-1 : 2n-1$.
- 39.) The sum upto n terms of the series : $1 + (1+x)y + (1+x+x^2)y^2 + (1+x+x^2+x^3)y^3 + \dots$
- 40.) Find the value of n so that $[a^{(n+1)} + b^{(n+1)}] / [a^n + b^n]$ may be the geometric mean between a and b .
- 41.) If $a(1/b + 1/c)$, $b(1/c + 1/a)$, $c(1/a + 1/b)$ are in A.P, prove that a, b, c are in A.P.
- 42.) If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, prove that x, y, z are in A.P.
- 43.) Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the balance in annual instalment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?